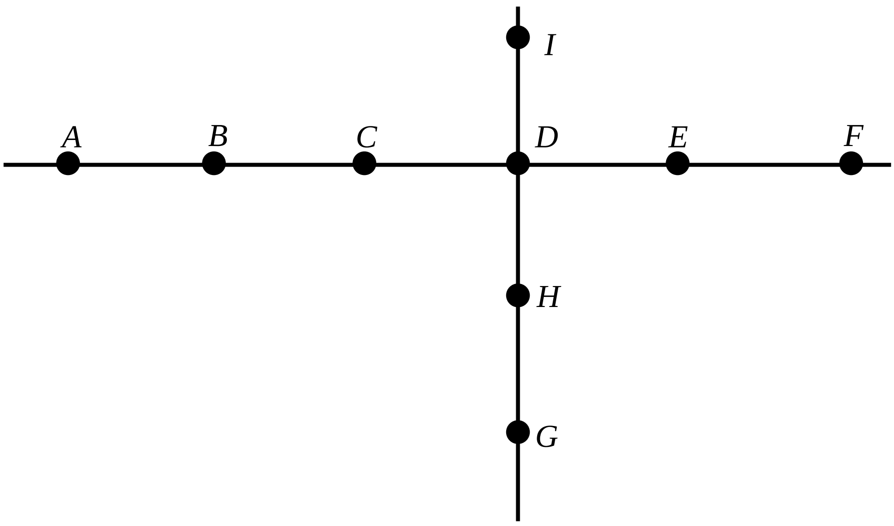
Hong Kong Mathematics Olympiad (2016/17)
Heat Event (Group)
香港数学竞赛 (2016/17)
初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 设 ΔABC 为一个等腰直角三角形，顶点 A 及 B 的坐标分别为 (–2, 0) 及 (18, 0)，且 C 的坐标是正数。当 ΔABC 的面积为最小时，求 C 的坐标。
Suppose that ΔABC is an isosceles right-angled triangle with the coordinates of the vertices A and B as (–2, 0) and (18, 0), respectively, and the coordinates of C having positive values. Determine the coordinate of C when the area of ΔABC attains its minimum.

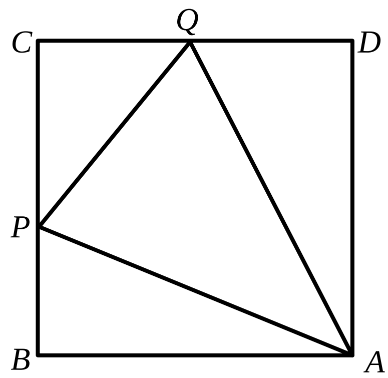
2. 如图一所示，点 A, B, C, D, E 及 F 均在一直在线，点 G, H, D 及 I 在另一直在线。拣选三点，可形成多少个三角形?
As shown in Figure 1, points A, B, C, D, E and F lie on the same straight line, and points G, H, D, and I lie on another straight line. How many triangles can be made by connecting any three points?



图一
Figure 1

3. 如图二所示，P、Q 分别是正方形 $ABCD$ 的边 BC 及 CD 上的点。已知ΔPCQ 的周界的长等于正方形 $ABCD$ 的周界的长的 $\frac{1}{2}$，求 $\angle PAQ$。

As shown in Figure 2, P , Q are points on the sides BC and CD of a square $ABCD$. Given that the perimeter of ΔPCQ is $\frac{1}{2}$ of that of the square $ABCD$, find $\angle PAQ$.

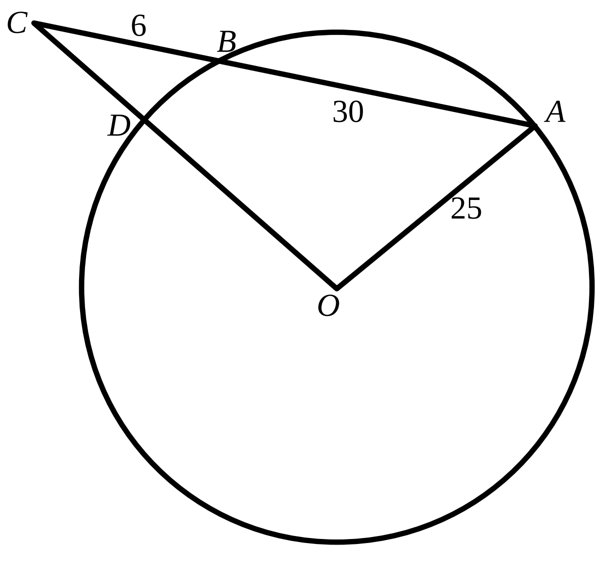


图二

Figure 2

4. 在图三中，O 是圆心。弦 AB 及半径 OD 的延线相交于 C。已知$OA = 25$、 $AB = 30$ 及 $BC = 6$。求 CD 的长。

In Figure 3, O is the centre of the circle. Chord AB and radius OD are produced to meet at C. Given that $OA = 25$, $AB = 30$ and $BC = 6$, find the length of CD.



图三

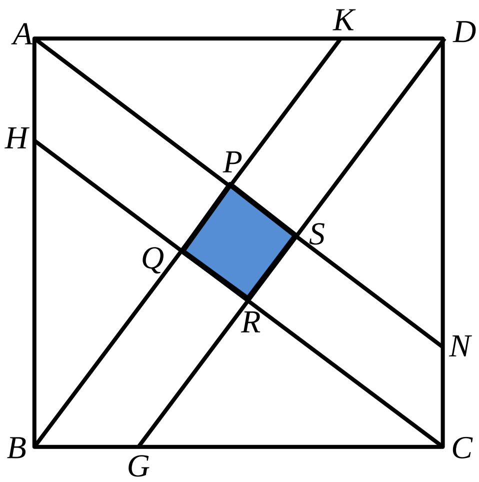
Figure 3

5. 设 Q 为所有能满足不等式 $\dfrac{9p^2}{\left(\sqrt{3p+1}-1\right)^2} < 3p+10$ 的整数 p 之和，求 Q 的值。

Let Q be the sum of all integers p satisfying the inequality $\dfrac{9p^2}{\left(\sqrt{3p+1}-1\right)^2} < 3p+10$, find the value of Q.

6. 在图四中，正方形 $ABCD$ 的边长为 20。 已知 $DK : KA = AH : HB = 1 : 3$ 及 BK // GD，HC // AN，求阴影部分 $PQRS$ 的面积。

In Figure 4, square $ABCD$ has sides of length 20. Given that $DK : KA = AH : HB = 1 : 3$ and BK // GD, HC // AN, find the area of the shaded region $PQRS$.



图四

Figure 4

7. 已知对于实数 $x_1, x_2, x_3, \cdots, x_{2017}$,

$$\sqrt{x_1-1}+\sqrt{x_2-1}+\sqrt{x_3-1}+\cdots+\sqrt{x_{2017}-1}=\frac{1}{2}\left(x_1+x_2+x_3+\cdots+x_{2017}\right).$$

求 $x_1+x_2+x_3+x_4+\cdots+x_{2017}$ 的值。

It is given that for real numbers $x_1, x_2, x_3, \cdots, x_{2017}$,

$$\sqrt{x_1-1}+\sqrt{x_2-1}+\sqrt{x_3-1}+\cdots+\sqrt{x_{2017}-1}=\frac{1}{2}\left(x_1+x_2+x_3+\cdots+x_{2017}\right).$$

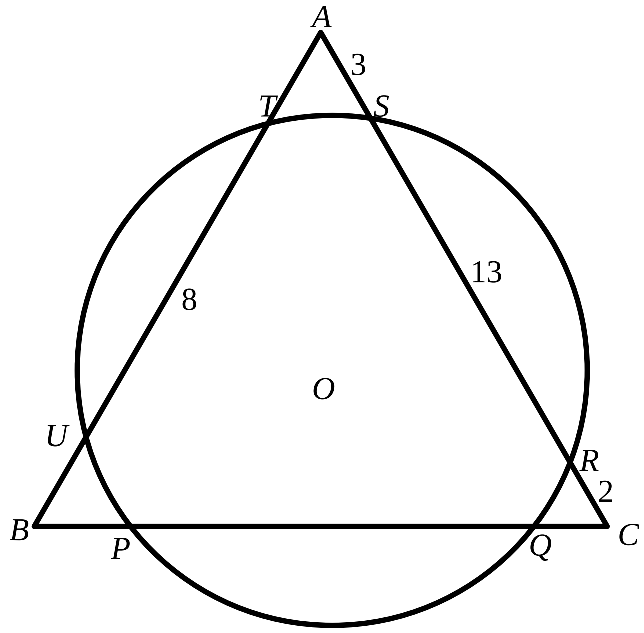
Find the value of $x_1+x_2+x_3+x_4+\cdots+x_{2017}$.

8. 设正整数 T 能满足条件：T 的数字的积$=T^2-11T-23$。求该等正整数之和，S，的值。

Let positive integers, T, satisfy the condition: the product of the digits of $T=T^2-11T-23$. Find the sum, S, of all such positive integers.

9. 在图五中，ABC 是一个等边三角形且与一圆相交于六点，P、Q、R、S、T 及 U。若 $AS=3$，$SR=13$，$RC=2$ 及 $UT=8$，求 $BP-QC$ 的值。

In Figure 5, ABC is an equilateral triangle intersecting the circle at six points P, Q, R, S, T and U. If $AS=3$, $SR=13$, $RC=2$ and $UT=8$, find the value of $BP-QC$.



图五

Figure 5

10. 已知方程 $a^2x^2-\left(4a-3a^2\right)x+2a^2-a-21=0$（其中 $a>0$）有最少一个整数根，求所有 a 的可能整数值之和。

It is given that the equation $a^2x^2-\left(4a-3a^2\right)x+2a^2-a-21=0$ (where $a>0$) has at least one integer root. Find the sum of all possible integral values of a.

完

END